Quantum Error Correction and One-Way LOCC

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Abstract. We consider one-way LOCC measurement protocols as quantum channels on bipartite systems and apply the Knill-Laflamme conditions from quantum error correction, obtaining new ways to identify one-way LOCC-distinguishable and indistinguishable bases of a given subspace. In the case of maximally-entangled states, these results can be related to quantum teleportation. We then apply our results the class of lattice states, a natural generalization of the Bell states, to construct relatively small sets of maximally-entangled states that are indistinguishable by any one-way LOCC scheme. We also present examples applying this approach to sets of generalized Pauli matrices.

Keywords: LOCC, maximally-entangled states, quantum error-correction, quantum channels.

A fundamental problem in Quantum Information is that of recovering classical information that has been encoded in quantum systems. This becomes particularly challenging when the system consists of multiple subsystems which cannot physically interact with each other. In this case, measurements can only be implemented with local quantum operations and classical communication (LOCC). Here, we suppose that two parties Alice and Bob share an unknown pure state from a known set of states $\mathcal{S}$ on a shared finite-dimensional quantum system. Their task is to determine the state's identity using only one-way LOCC measurement operations: Alice will perform a local measurement on her system and report the results to Bob, who will complete the identification by performing a measurement on his subsystem.

The first step of an optimal one-way LOCC protocol can be viewed as a quantum-classical channel Φ acting on Alice's system, and the recovery of the elements of $\mathcal{S}$ is related to error correction of the channel on the subspace C spanned by the elements of $\mathcal{S}$. We apply the Knill-Laflamme conditions from quantum error correction to study this.

In the case that $\mathcal{S}$ is a set of d maximally-entangled states on $d\otimes d$, we show that distinguishability of the states in $\mathcal{S}$ implies the correctability of the entire subspace C. In this case, Alice's optimal measurement can be used as the first step in a teleportation protocol applied to an unknown state in C

The error-correction conditions also provide necessary conditions for one-way LOCC discrimination. We apply them to discover small sets of maximally entangled states that cannot be distinguished with one-way LOCC. Our examples are subsets of the lattice states and of the generalized Pauli states in $d\otimes d$, and each set has size less than $3\sqrt{d}$.

We frame these examples in the context of other small sets of maximally entangled states that can and cannot be distinguished with one-way LOCC.

References

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