# Detecting Bell-CHSH nonlocality of pure symmetric three-qubit states

**K. Anjali[1], Akshata Shenoy H.[2], H. S. Karthik[2], Shradhanjali Sahu[3], Sudha[4,5] and A. R. Usha Devi[1,5]**

*1 Department of Physics, Jnanabharathi, Bangalore University, Bangalore-560056, India.*

*2 International Centre for Theory of Quantum Technologies, University of Gdansk, Gdansk, Poland.*

*3 School of Electronics and Electrical Engineering, University of Leeds, Leeds, United Kingdom.*

*4 Department of Physics, Kuvempu University, Shankaraghatta-577 451, India.*

*5 Inspire Institute Inc., Alexandria, Virginia, 22303, USA.*

**Abstract:** We explore nonlocality of three-qubit pure symmetric states shared between Alice, Bob and Charlie using the Clauser-Horne-Shimony-Holt (CHSH) inequality. We show that the reduced two-qubit states, extracted from an arbitrary pure entangled symmetric three-qubit state, do not violate the CHSH inequality. However, when Alice and Bob perform the CHSH test, after conditioning over the measurement results of Charlie, nonlocality of the state is revealed. It is also shown that two different classes of the three-qubit pure symmetric states, consisting of two and three distinct constituent spinors, can be distinguished through the conditional CHSH nonlocality test.

**Keywords**: Nonlocality, Permutation symmetry, Majorana representation

Permutation symmetric multiqubit states have attracted a lot of attention in the field of quantum information processing due to their mathematical elegance, experimental ease [1,2] and applications [3,4,5,6,7,8,9]. We make extensive use of the Majorana geometrical representation which offers a natural classification of pure entangled symmetric three-qubit states in terms of two and three distinct points on the Bloch sphere. Further, these states possess an elegant parametrization for different SLOCC classes with respect to the Majorana representation [10,11]. In the present work, we carry out detailed investigations on the nonlocality of pure three-qubit symmetric states based on variants of the CHSH inequality [12,13,14]. In order to investigate the nonlocal content in terms of the Majorana class, we consider the reduced two qubit density matrices extracted from these states. They are found to be CHSH local. This prompts us to explore the conditional CHSH inequality [13,14] which is useful in bringing forth the hidden two-qubit nonlocality thereby indicating the nonlocal content of the entire state. Our observation reveals that the hidden nonlocality of two qubit permutation symmetric states is activated by conditioning on the outcomes of measurements performed on the third qubit [16].

The geometric representation of an arbitrary pure three-qubit symmetric state is given by

$$|\psi_{sym}\rangle = N\sum_{P}\hat{P}\{|\alpha_1,\beta_1\rangle|\alpha_2,\beta_2\rangle|\alpha_3,\beta_3\rangle\}$$

where $|\alpha_k,\beta_k\rangle \equiv cos\frac{\beta_k}{2}|0\rangle + e^{i\alpha_k}\,sin\frac{\beta_k}{2}|1\rangle$, $k = 1,2,3$. They can be classified into two ($\{D_{3,2}\}$ when $|\alpha_1,\beta_1\rangle = |\alpha_2,\beta_2\rangle \neq |\alpha_3,\beta_3\rangle$) and three ($\{D_{3,3}\}$ when $|\alpha_1,\beta_1\rangle \neq |\alpha_2,\beta_2\rangle \neq |\alpha_3,\beta_3\rangle$ ) distinct spinor classes. Under local unitary operations, the states in $\{D_{3,2}\}$ and $\{D_{3,3}\}$ reduce to $|\psi_{3,2}\rangle = N\sum_P \hat{P}\{|0\rangle|0\rangle|\beta\rangle\}$ and $|\psi_{3,3}\rangle = N\{|0\rangle^{\otimes 3} + y\,e^{i\alpha}|\beta\rangle^{\otimes 3}\}$. Here, $|\beta\rangle = cos\frac{\beta}{2}|0\rangle + sin\frac{\beta}{2}|1\rangle$, $0<\beta\leq\pi, 0<y\leq 1, 0\leq\alpha\leq 2\pi$. It is seen that $\{D_{3,2}\}$ and $\{D_{3,3}\}$ class is characterized by one ($\beta$) and three parameters ($\beta,\alpha,y$) respectively. We make use of this explicit parametrization [11, 15] to study their nonlocality [16].

The conditional CHSH inequality employed here is, $\left|\sum_{c\pm 1}(\langle A_1B_1\rangle_c + \langle A_1B_2\rangle_c + \langle A_2B_1\rangle_c - \langle A_2B_2\rangle_c)\right| \leq 2$ where $\langle A_iB_j\rangle_c = p(c)\sum_{a_i,b_j=\pm 1} a_i, b_j\, p\big(a_i,b_j|A_i,B_j,c\big)$. Conditioned on Charlie's outcome $c = \pm 1$ for his measurement $P_c$, Alice, Bob measure $A_i$, $B_j$, i=j=1,2 to obtain dichotomic outcomes $a_i, b_j = \pm 1$.

**Results:**

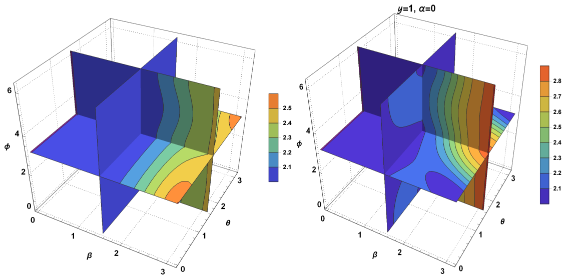


**Figure1** Slice contour plot for $\{D_{3,2}\}$(left) and $\{D_{3,3}\}$(right) class as a function of the state parameter $\beta$ and Charlie's measurement orientation angles $\theta,\phi$. In case of $\{D_{3,3}\}$ class, y=1 and $\alpha = 0$.

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