# Quantum computation from the linear-optical dynamics of bosonic anyons

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**Abstract.** We define a computational model that uses a one-dimensional lattice of anyons defined by deformed bosonic and fermionic commutation relations. We solve the nonlinear dynamics of the quadratic, number-preserving Hamiltonians describing optical elements, and show that the model is universal for quantum computation. For the case of deformed bosonic statistics, we describe the appropriate generalizations of typical bosonic behavior, such as the Hong-Ou-Mandel effect and coherent-state dynamics.

**Keywords**: Anyons, Optics, Quantum Computation, Quantum Simulation

Anyons were first studied as a generalization of quantum statistics for two-dimensional systems [1-3], but it has been shown there are many non-equivalent ways to define anyonic statistics also in one dimension [1,4]. One particular definition has been extensively investigated in the optical lattice community [5,6] due to progress in simulations of density-dependent and gauge interactions between atoms in optical lattices [7,8].

In this work, we analyze these anyonic models to define a computing model based on the linear optics of these systems [9]. The search for analogies with linear optics to understand the computing power of identical particles is not new [10], and it has shown many bridges between qubit-based models of restricted computation, such as *Matchgates* [11] with "optical like" models such as *Fermionic Linear Optics* [12,13].

Therefore, our objective is to give a generalization of these relations and to show the differences between each model which are due to the statistical character of the particles.

The anyonic models that we study result from deformations of the usual bosonic and fermionic commutation relations, defining what we call bosonic and fermionic anyon models. In both cases, the anyonic character is defined only by the value of the exchange phase ($-e^{i\varphi}$ for fermionic anyons and $e^{i\varphi}$ for bosonic anyons) acquired when we exchange two anyons.

For fermionic anyons, we show in [14] that for all values of the exchange phase except for usual fermions, the corresponding model is universal for quantum computing. This requires interactions between non-nearest neighbor modes and is explained by the appearance of phases induced by statistics.

For bosonic anyons, we prove in [15] that, similarly, all non-trivial models are universal for quantum computation. We also prove that the Hong-Ou-Mandel bunching effect is still present, with the added appearance of exchange phases. We show these phases are the mechanism providing computational universality. We also study how to define coherent states for these models, showing that exchange phases play an important role in their behavior under the action of simple optical elements.

Bosonic and fermionic linear optics are very different in computational power. The former is hard to simulate on a classical computer [16], whereas the latter is efficiently simulable [12]. Interestingly, our work shows that this distinction does not persist under the deformation of statistics that we consider, since both particle types immediately become universal for quantum computation for any nontrivial values of the statistical phase.

**References**

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