Detecting entanglement of unknown continuous-variable states with random measurements

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Abstract. We explore the possibility of entanglement detection in continuous-variable systems by entanglement witnesses based on covariance matrices, constructible from random homodyne measurements. We propose new linear constraints characterising the entanglement witnesses based on second moments, and use them in a semidefinite program providing the optimal entanglement test for given random measurements. We test the method on the class of squeezed vacuum states and study the efficiency of entanglement detection in general unknown covariance matrices.

Keywords: continuous-variable entanglement, entanglement witness, semidefinite programming, homodyne detection.

A significant problem in quantum information theory is to efficiently reveal the properties of an unknown quantum state, for instance the presence of entanglement.
For continuous-variable (CV) systems, the usual entanglement criteria, such as the positive partial transpose (PPT) criterion, consist in certain operations on the covariance matrix (the matrix of the second moments) of the states [1,2].
For a completely unknown state, a full tomography is then required to reconstruct entirely its covariance matrix, but this method may be a very resource-consuming and demanding experimental procedure.
Besides, the full information about the second moments of a state can be excessive for the characterisation of its entanglement.
Entanglement witnesses (EWs) represent a commonly used entanglement test, being directly accessible in experiments through measurable observables [3].
Given an unknown quantum state, however, the complexity of the state and the absence of any information about it deprive us of a specific experimental strategy in tackling the problem of efficient entanglement detection.
We propose a method to detect entanglement of unknown CV states, given only the dimension of their covariance matrices.
We use entanglement witnesses based on second moments, constructible from random homodyne measurements. This idea is inspired by an analogous method for the discrete-variable case [4].
We derive new linear constraints characterising the witnesses, and use them in a semidefinite program providing the optimal entanglement test for given random measurements.
We study the feasibility of this method in experimental situations, where the figure of merit is considered the number of measurements required to detect entanglement.
First, we test the proposed algorithm for two-mode squeezed vacuum states, for which the logarithmic negativity linearly depends on the squeezing. Then, we simulate the performance of this method for uniformly drawn random two-mode covariance matrices, without adding any information about the states.
We show that the number of necessary random measurements is very likely to be smaller than for a full tomography, hence proving the validity of our method.

References

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