

Robustness of Noether's principle: Maximal disconnects between conservation laws and symmetries in quantum theory [1]

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Noether's theorem connects symmetries with conservation laws and plays a key role in the development of modern physics. The applicability of traditional formulations of Noether's theorem are, however, mostly restricted to isolated classical and quantum systems, raising the fundamental question: *to what extent Noether's principle holds for open (non-isolated) quantum systems?* In [1] we investigate how robust are conservation laws under a dynamics described by a symmetric quantum channel. The analysis required to extend previous structural results on the convex set of symmetric operations along with new results on decoherence measures. These play a key role to derive quantitative trade-off relations between decoherence and violations of conservation laws for systems that undergo a symmetric general quantum process. We obtained bounds on the unitarity of a symmetric channel in terms of the average deviation from conservation laws for charge operators that generate the symmetry principle. Consequently, we show that if a symmetric quantum channel approximates a symmetric unitary dynamics then the corresponding conservation law holds approximately. The converse occurs only for particular cases such as spin systems carrying an irreducible representation of $SU(2)$, and more generally whenever the input and output operator spaces have a multiplicity-free decomposition into irreducible components. Therefore, these particular types of symmetries lead to a robustness of conservation laws under symmetric interaction with an environment. Furthermore, the structure of quantum theory places constraints on the maximal disconnects between conservation laws and symmetries. Using the characterisation of extremal points of the set of $SU(2)$ symmetric quantum operations we obtain the maximal expected deviations from a conservation law of spin- j systems. We show that these fundamental limitations are related to physically impossible operations such as time-reversal and spin-inversion and find the limits on how much spin systems can be amplified or inverted under a rotationally symmetric process.

To conclude, our main contribution is to quantitatively capture the discrepancy between conservation laws and symmetries for irreversible dynamics via decoherence measures that are experimentally accessible using randomised benchmarking (i.e unitarity). Therefore, this work is relevant to a range of applications from symmetry checking for simulations and symmetry-analysis for benchmarking of quantum devices to the development of universal bounds for the thermodynamics of quantum systems.

References

- [1] Cîrstoiu C, Korzekwa K, Jennings D., *Robustness of Noether's principle: Maximal disconnects between conservation laws and symmetries in quantum theory*, ArXiv preprint arXiv:1908.04254 <https://arxiv.org/abs/1908.04254> .