Bell nonlocality with a single shot

Mateus Araújo[1], Flavien Hirsch[1], and Marco Túlio Quintino[2]

[1]Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria

[2]Department of Physics, Graduate School of Science, The University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan

31st July 2020

In order to reject the local hidden variables hypothesis, the usefulness of a Bell inequality can be quantified by how small a p-value it will give for a physical experiment. When the Bell inequality is reformulated as a nonlocal game with local bound ω_ℓ and Tsirelson bound ω_q, we show that the expectation value of its p-value after n rounds is upperbounded by $(1-(\omega_q-\omega_\ell)^2)^n$, based on the results of [1]. Therefore, having a large gap $\omega_q-\omega_\ell$ implies having a small expected p-value, and the gap is a useful figure of merit to optimise. We develop an algorithm for transforming an arbitrary Bell inequality into such an optimal nonlocal game, showing that it reduces to solving a linear programming problem, and present its results for the CGLMP and I_{nn22} inequalities.

We also present explicit examples of nonlocal games such that the gap between their local and Tsirelson bounds is arbitrarily close to one. Since this implies that the probability of winning the nonlocal game with the optimal quantum strategy is arbitrarily close to one, and the p-value of such a victory is arbitrarily close to zero, this makes it possible to reject local hidden variables with arbitrarily small p-value in a single shot, without needing to collect statistics.

The first example consists of playing n copies of the CHSH game simultaneously, its *parallel repetition.* Using Rao's bound [2] we show that the probability of winning more than $3/4$ of the parallel instances, the amount expected from its local bound, goes to zero exponentially with n. On the other hand, the quantum probability of winning more than $3/4$ of the parallel instances, but fewer than $(2+\sqrt{2})/4$ of them, goes exponentially to 1, allowing us to obtain an arbitrarily small p-value. As an example, to achieve a p-value of 10^{-5} it is enough to have 67 683 296 parallel instances, or a quantum state of local dimension $2^{67\,683\,296}$.

The second example demonstrates that parallel repetition is not necessary to obtain a single-shot rejection of local hidden variables: it consists of the Khot-Vishnoi game [3–5] with a choice of parameters such that its local bound is upperbounded by $\frac{1}{\log(\sqrt[4]{d})}$ and its Tsirelson bounded lowerbounded by $1-\frac{\log(\log(\sqrt[4]{d}))}{\log(\sqrt[4]{d})}$. Here d is both the number of outputs per party and the local dimension of the quantum state used to achieve this quantum probability of success. It then follows that to achieve a p-value of 10^{-5} it is enough to have a quantum state of local dimension $2^{577\,079}$. This dimension is much smaller than in the CHSH case, but it does not imply a simpler experimental setup, because to obtain this probability of success in the Khot-Vishnoi game one needs to implement entangled measurements on the whole quantum state, whereas in the CHSH case independent measurements suffice.

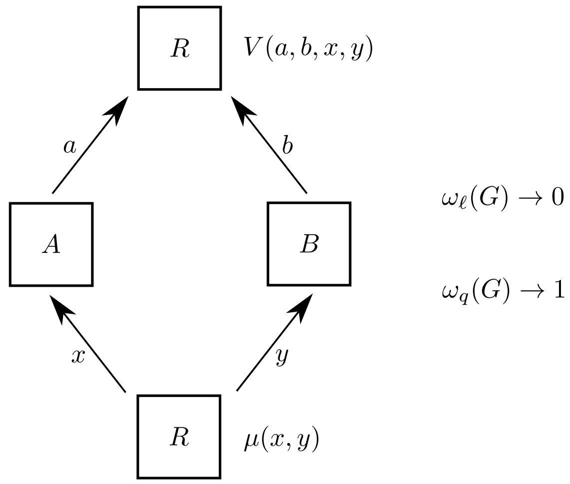

Figure 1: Schematic representation of a bipartite nonlocal game. A referee samples questions x and y with probability $\mu(x,y)$ and sends them to Alice and Bob. They send their answers a and b back to the referee, who accepts their answers with probability $V(a,b,x,y)$, in which case they win. If the maximal probability of winning the game with local hidden variables $\omega_\ell(G)$ is close to zero and the probability of winning it with the optimal quantum strategy $\omega_q(G)$ is close to one then this nonlocal game makes it possible to reject local hidden variables in a single round.

This raises the question of whether it is possible to achieve a single-shot rejection of local hidden variables with easier experimental setups. To answer that, we considered what is the largest possible gap that can be achieved using quantum states of local dimension d, and showed that $\omega_q-\omega_\ell \leq 1-\frac{3}{ed}$, based on the results of [6, 7]. If there exists a nonlocal game achieving this bound, a quantum system of local dimension 2^{17} would be enough for the single-shot rejection.

References

[1] D. Elkouss and S. Wehner. '(Nearly) optimal P-values for all Bell inequalities'. *npj Quantum Information* **2**, 16026 (2016). arXiv:1510.07233 [quant-ph].

[2] A. Rao. 'Parallel repetition in projection games and a concentration bound'. *SIAM Journal on Computing* **40** 1871–1891 (2011).

[3] S. A. Khot and N. K. Vishnoi. 'The unique games conjecture, integrality gap for cut problems and embeddability of negative type metrics into ℓ_1'. *46th Annual IEEE Symposium on Foundations of Computer Science (FOCS'05)*. 2005, pp. 53–62. arXiv:1305.4581 [cs.CC].

[4] J. Kempe, O. Regev and B. Toner. 'Unique Games with Entangled Provers are Easy' (2007). arXiv:0710.0655 [quant-ph].

[5] H. Buhrman, O. Regev, G. Scarpa and R. de Wolf. 'Near-Optimal and Explicit Bell Inequality Violations'. *2011 IEEE 26th Annual Conference on Computational Complexity* (2011). arXiv:1012.5043 [quant-ph].

[6] M. L. Almeida, S. Pironio, J. Barrett, G. Tóth and A. Acín. 'Noise Robustness of the Nonlocality of Entangled Quantum States'. *Physical Review Letters* **99** 040403 (2007). arXiv:quant-ph/0703018.

[7] C. Palazuelos. 'On the largest Bell violation attainable by a quantum state' (2012). arXiv:1206.3695 [quant-ph].