Convex Gaussian resource theories and resource distillations

Hyejung H. Jee[1], Carlo Sparaciari[1,2], and Mario Berta[1]

[1]Departmet of Computing, Imperial College London, London SW7 2AZ, UK
[2]Department of Physics and Astronomy, University College London, London WC1E 6BT, UK

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Abstract It is well-known that resource distillation in general Gaussian resource theories is impossible. To overcome this limitation, we enlarge the theories to include convex mixtures of Gaussian states and operations. This extension is operationally well-motivated since classical randomness is easily accessible in a laboratory. We find that resource distillation becomes possible for convex Gaussian resource theory (albeit in a limited fashion), and we provide explicit protocols for entanglement and squeezing theory. We fully characterise the new limitations by introducing a new general measure of resource applicable to convex mixtures of Gaussian states, and we derive several property of this monotone.

Keywords Quantum resource theories, resource distillation, convex hull of Gaussian states.

The Gaussian framework is a powerful tool for exploring the features of quantum theory, but several limitations on quantum information processing tasks are known to hold for this framework. For instance, no quantum advantage is possible with Gaussian quantum computing [1–3], Gaussian error correction is impossible [4] as well as Gaussian entanglement distillation [5, 6]. Recently, Gaussian resource theories were formally introduced in [7]; the authors describe Gaussian resource theories as quantum resource theories which are further restricted to Gaussian states and operations. Within this framework, they show that distillation of general resources (entanglement being a prominent example) is impossible.

In our paper, we examine how the non-convex structure of the set of Gaussian states contributes to the impossibility of resource distillation in Gaussian resource theories. We first introduce the notion of convex Gaussian resource theories, where the state space is composed by the convex hull of Gaussian states, and we look at resource distillations in such theories. The inclusion of convex mixtures of Gaussian states is well-motivated from an operational point of view, since it is not difficult to create such states in the laboratory using Gaussian states and classical randomness.

We find that conditional Gaussian operations can be used to perform a (somewhat limited) resource distillation from mixtures of Gaussian states, while still preserving the set of free states. This is in contrast with the fully Gaussian case, where conditional, or probabilistic, resource distillation is proven to be impossible [7]. To show that distillation is possible for convex resource theories, we provide explicit protocols for both entanglement and squeezing theory, building on previous results from [8, 9].

Finally, we are able to fully characterise the limitations affecting resource distillation in general convex Gaussian resource theories. Specifically, we show that the output state of any distillation protocol cannot be more resourceful than the most resourceful state contained in the initial mixture, even when many copies of the mixture are available. To prove this no-go result, we introduce a new measure of general resources for convex mixtures of Gaussian states. This is the convex-roof extension of another measure, valid only for Gaussian states, that was introduced in [7]. We prove several properties for this measure, such as monotonicity under conditional Gaussian operations, convexity, and faithfulness.

Our results show that convex Gaussian resource theories, while being operationally meaningful, are still subject to strong limitations for distillation. This result seems to further indicate that resource distillation within Gaussian theory can only be fully achieved when more exotic kind of non-Gaussian states (or operations) are allowed, such as for instance those with a negative Wigner function.

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