Characterising quantum correlations of fixed dimension

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Abstract We give a converging semidefinite programming hierarchy of outer approximations for the set of quantum correlations of fixed dimension. Starting from the Navascués-Pironio-Acín (NPA) hierarchy, we identify additional semidefinite constraints for any fixed dimension, which lead to analytical bounds on the convergence speed. In particular, we give an algorithm to compute additive ε-approximations on the value of two-player free games of $|T| \times |T|$-dimensional quantum assistance in time which scales polynomially in $|Q|$ and quasi-polynomially in $|A|$, where $|Q|$ and $|A|$ are the numbers of questions and answers, respectively. This improves previously known approximation algorithms which were at best exponential in $|Q||A|$.

Keywords non-local game, semidefinite programming, quantum correlation, approximation algorithm.

Computing the violation of locality constraints for quantum systems of bounded or unbounded dimension has been of great interest to both quantum physicists and computer scientists as it is interesting for both its fundamental [1] and practical [2] implications. For the case of unbounded dimension, the NPA hierarchy provides effective tools to calculate upper bounds on the violation of locality constraints [3], which has been widely employed in quantum information theory. There have been a few results for the case of bounded dimension [4–6], but the asymptotic convergence speed for these works is either not analytically quantified or at best exponential in $|Q||A|$.

In our work, we identify semidefinite constraints that can be added to the NPA hierarchy to impose dimension constraints. Along with the NPA constraints, they construct a new semidefinite programming (SDP) hierarchy of outer approximations for fixed-dimensional quantum correlations. As we add more constraints to the NPA hierarchy, the new hierarchy is guaranteed to produce at least as good results as the NPA hierarchy. We derive analytical bounds on the convergence speed of the new relaxations, which leads to upper bounds on the computational complexity of calculating the quantum value of two-player free games with $|T| \times |T|$-dimensional quantum assistance. Namely, we provide an approximation algorithm which can estimate the quantum value of two-player free games of dimension $|T| \times |T|$ within additive error ε in time

$$\exp\left(\mathscr{O}\left(\frac{|T|^8}{\varepsilon^2}\log|A||T|(\log|Q| + \log|A||T|)\right)\right),$$

where $|A|$ and $|Q|$ denote the numbers of answers and questions, respectively. Here, free games means that the questions for two players are not correlated. For fixed $|T|$, the dependence is quasi-polynomial in $|A|$ and polynomial in $|Q|$, which improves previously known approximation algorithms [4]. Our result is the quantum extension of the quasi-polynomial time approximation scheme for computing the classical value of free games [7, 8]. We also give an algorithm for general games, but it has a worse asymptotic convergence speed which is still quasi-polynomial in $|A|$ but exponential in $|Q|$.

Our approach for deriving the SDP relaxations is to make a connection to the quantum separability problem, following ideas somewhat similar to [4]. The main tool to quantify the convergence speed is an improved multipartite quantum de Finetti theorem with linear constraints. The technique to prove the convergence rate of the SDP relaxations using quantum de Finetti theorem with linear constraints is inspired from [7, 9].

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