

Witnessing quantum memory in non-Markovian processes

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Abstract. We present a method to detect quantum memory in a non-Markovian process. We call a process Markovian when the environment does not provide a memory that retains correlations across different system-environment interactions. We define two types of non-Markovian processes, depending on the required memory being classical or quantum. We formalise this distinction using the process matrix formalism, through which a process is represented as a multipartite state. Within this formalism, a test for entanglement in a state can be mapped to a test for quantum memory in the corresponding process. This allows us to apply separability criteria and entanglement witnesses to the detection of quantum memory. We demonstrate the method in a simple model where both system and environment are single interacting qubits and map the parameters that lead to quantum memory. As with entanglement witnesses, our method of witnessing quantum memory provides a versatile experimental tool for open quantum systems.

Keywords: quantum memory, non-markovianity, process matrix, non-markovian noise

In any quantum device, the system that carries the information unavoidably interacts with its environment introducing noise. Studying the dynamics of such system-environment interactions is the field of open quantum systems [1] and it is nowadays more relevant than ever. As quantum devices begin to demonstrate an advantage over classical ones [2], they increasingly rely on Noisy Intermediate-Scale Quantum (NISQ) technology, whose main challenge is noise [3].

Noise models typically rely on the assumption of Markovianity, i.e., that the environment does not keep memory of past interactions with the system. However, this assumption typically fails in realistic scenarios, as information stored in the environment can keep track of past interactions with the system and affect its future dynamics. For example, this was demonstrated to occur in the IBM quantum computing platform [4]. In the study of such memory effects, an important distinction is whether the memory can be represented classically or requires genuinely quantum degrees of freedom. The two scenarios can lead to radically different noise models and strategies to compensate it. It is therefore desirable to find efficient methods to discriminate quantum vs classical memory.

Most models of open quantum systems with memory regard the process as a dynamical map which maps the system from one time-step to the other [5–14]. Within this approach, many models of processes with classical memory have been developed [15–24]. However, dynamical maps study only two-time correlations (time of input and output of the map) and multi-time-correlations cannot be fully captured. Furthermore, dynamical maps are in general ill-defined in the presence of initial system-environment correlations [25–28], although such correlations can be responsible for non-Markovianity.

Here, we introduce a definition of quantum process with classical memory based on an approach that captures multi-time correlations, originally introduced by Lindblad [29] and Accardi et al. [30], and recently reformulated within the comb formalism [31] by Pollock et al. [32], Fig. 1. We provide a technique to

efficiently detect the presence of quantum memory in a non-Markovian process, without requiring full tomography. We use the process matrix formalism [33, 34] to write the process as a multipartite state. For a specific partition of the state, classical memory implies separability, while entanglement proves quantum memory. Therefore, we can employ all the known techniques that verify entanglement and use them to prove non-Markovianity with quantum memory.

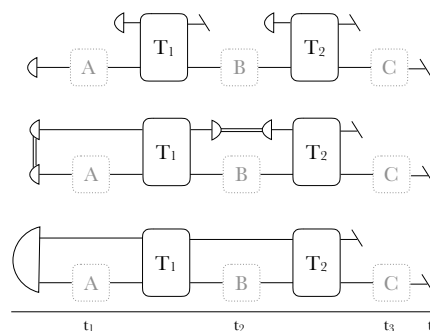


Figure 1. Three types of processes with three time-steps: Markovian (top), where the environment has no memory, classical memory (middle), where classical information from the system is carried by the environment, and quantum memory (bottom), where there are initial quantum correlations that travel across the process. A, B and C are places for general operations for process tomography.

To illustrate our method of detecting quantum memory, we use entanglement witnesses to obtain witnesses for quantum memory for the following toy model: system and environment are qubits jointly prepared in a maximally entangled state and later interact according to the Ising model, in between two measurement stations A and B for the system. A quantum memory witness corresponds to a set of operations at A and B. As separability criteria for the search of witnesses we use the positive partial transpose (PPT) applied on the state [35] and on symmetric extensions of the state [36]. To find a witness, we cast each criterion as a SemiDefinite Program that can be solved efficiently. This also allows us to restrict the search for witnesses, possibly tailored to experimental limitations. Preprint at <https://arxiv.org/abs/1811.03722>.

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